Rate of change is a ratio that describes how one quantity is changing with respect to another. Unit rate is a rate that compares the change in one quantity to a one-unit change in another quantity. Some examples of rates are miles per hour and price per pound. If 16 ounces of flour cost $0.80 then the unit cost, that is the cost per one once, is $\frac{\$0.80}{16} = \$0.05$.

For additional information see the Math Notes boxes in Lesson 7.1.3 of the *Core Connections, Course 1* text, Lesson 4.2.4 of the *Core Connections, Course 2* text, or Lesson 7.2.5 of the *Core Connections, Course 3* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 9 materials or the *Core Connections, Course 3* Checkpoint 3 materials.

Example 1

A rice recipe uses 6 cups of rice for 15 people. At the same rate, how much rice is needed for 40 people?

The rate is: $\frac{6 \text{ cups}}{15 \text{ people}}$ so we need to solve $\frac{6}{15} = \frac{x}{40}$.

The multiplier needed for the Giant One is $\frac{40}{15}$ or $2\frac{2}{3}$.

Using that multiplier yields $\frac{6}{15} \cdot \frac{2\frac{2}{3}}{2\frac{2}{3}} = \frac{16}{40}$ so 16 cups of rice is needed.

Note that the equation $\frac{6}{15} = \frac{x}{40}$ can also be solved using proportions.

Example 2

Arrange these rates from least to greatest:

30 miles in 25 minutes

60 miles in one hour

70 miles in $1\frac{2}{3}$ hr

Changing each rate to a common denominator of 60 minutes yields:

$\frac{30 \text{ mi}}{25 \text{ min}} = \frac{x}{60} \Rightarrow \frac{30}{25} \cdot \frac{2.4}{2.4} = \frac{72}{60} \frac{\text{mi}}{\text{min}}$

$\frac{60 \text{ mi}}{1 \text{ hr}} = \frac{60 \text{ mi}}{60 \text{ min}}$

$\frac{70 \text{ mi}}{1\frac{2}{3} \text{ hr}} = \frac{70 \text{ mi}}{100 \text{ min}} = \frac{x}{60} \Rightarrow \frac{70}{100} \cdot \frac{0.6}{0.6} = \frac{42 \text{ mi}}{60 \text{ min}}$

So the order from least to greatest is: 70 miles in $1\frac{2}{3}$ hr < 60 miles in one hour < 30 miles in 25 minutes. Note that by using 60 minutes (one hour) for the common unit to compare speeds, we can express each rate as a unit rate: 42 mph, 60 mph, and 72 mph.

Example 3

A train in France traveled 932 miles in 5 hours. What is the unit rate in miles per hour?

Unit rate means the denominator needs to be 1 hour so: $\frac{932 \text{ mi}}{5 \text{ hr}} = \frac{x}{1 \text{ hr}}$. Solving by using a Giant One of $\frac{0.2}{0.2}$ or simple division yields $x = 186.4$ miles per hour.

Problems

Solve each rate problem below. Explain your method.

1. Balvina knows that 6 cups of rice will make enough Spanish rice to feed 15 people. She needs to know how many cups of rice are needed to feed 135 people.

2. Elaine can plant 6 flowers in 15 minutes. How long will it take her to plant 30 flowers at the same rate?

3. A plane travels 3400 miles in 8 hours. How far would it travel in 6 hours at this rate?

4. Shane rode his bike for 2 hours and traveled 12 miles. At this rate, how long would it take him to travel 22 miles?

5. Selina's car used 15.6 gallons of gas to go 234 miles. At this rate, how many gallons would it take her to go 480 miles?

6. Arrange these readers from fastest to slowest: Abel read 50 pages in 45 minutes, Brian read 90 pages in 75 minutes, and Charlie read 175 pages in 2 hours.

7. Arrange these lunch buyers from greatest to least assuming they buy lunch 5 days per week: Alice spends $3 per day, Betty spends $25 every two weeks, and Cindy spends $75 per month.

8. A train in Japan can travel 813.5 miles in 5 hours. Find the unit rate in miles per hour.

9. An ice skater covered 1500 meters in 106 seconds. Find his unit rate in meters per second.

10. A cellular company offers a price of $19.95 for 200 minutes. Find the unit rate in cost per minute.

11. A car traveled 200 miles on 8 gallons of gas. Find the unit rate of miles per gallon and the unit rate of gallons per mile.

12. Lee's paper clip chain is 32 feet long. He is going to add paper clips continually for the next eight hours. At the end of eight hours the chain is 80 feet long. Find the unit rate of growth in feet per hour.

Answers

1. 54 cups   2. 75 min   3. 2550 miles   4. $3\frac{2}{3}$ hr

5. 32 gallons   6. C, B, A   7. C, A, B   8. 162.7 mi/hr

9. ≈ 14.15 m/s   10. $\approx \$0.10$/min   11. 25 m/g; $\frac{1}{25}$ g/m   12. 6 ft/hr

DISTANCE, RATE, AND TIME

Distance (d) equals the product of the rate of speed (r) and the time (t). This relationship is shown below in three forms:

$$d = r \cdot t \qquad r = \frac{d}{t} \qquad t = \frac{d}{r}$$

It is important that the units of measure are consistent. For additional information see the Math Notes box in Lesson 8.3.2 of the *Core Connections, Course 1* text.

Example 1

Find the rate of speed of a passenger car if the distance traveled is 572 miles and the time elapsed is 11 hours.

$$572 \text{ miles} = r \cdot 11 \text{ hours} \implies \frac{572 \text{ miles}}{11 \text{ hours}} = r \implies 52 \text{ miles/hour} = \text{rate}$$

Example 2

Find the distance traveled by a train at 135 miles per hour for 40 minutes.

The units of time are not the same so we need to change 40 minutes into hours. $\frac{40}{60} = \frac{2}{3}$ hour.

$$d = (135 \text{ miles/hour})(\tfrac{2}{3} \text{ hour}) \implies d = 90 \text{ miles}$$

Example 3

The Central Middle School hamster race is fast approaching. Fred said that his hamster traveled 60 feet in 90 seconds and Wilma said she timed for one minute and her hamster traveled 12 yards. Which hamster has the fastest rate?

rate $= \frac{distance}{time}$ but all the measurements need to be in the same units. In this example, we use feet and minutes.

Fred's hamster: rate $= \frac{60 \text{ feet}}{1.5 \text{ minutes}}$ $\Rightarrow$ rate $= 40$ feet/minute

Wilma's hamster: rate $= \frac{36 \text{ feet}}{1 \text{ minute}}$ $\Rightarrow$ rate $= 36$ feet/minute

Fred's hamster is faster.

Problems

Solve the following problems.

1. Find the time if the distance is 157.5 miles and the speed is 63 mph.

2. Find the distance if the speed is 67 mph and the time is 3.5 hours.

3. Find the rate if the distance is 247 miles and the time is 3.8 hours.

4. Find the distance if the speed is 60 mph and the time is 1 hour and 45 minutes.

5. Find the rate in mph if the distance is 3.5 miles and the time is 20 minutes.

6. Find the time in minutes if the distance is 2 miles and the rate is 30 mph.

7. Which rate is faster? A: 60 feet in 90 seconds or B: 60 inches in 5 seconds

8. Which distance is longer? A: 4 feet/second for a minute or B: 3 inches/min for an hour

9. Which time is shorter? A: 4 miles at 60 mph or B: 6 miles at 80 mph

Answers

1.   2.5 hr   2.   234.5 mi   3.   65 mph   4.   105 miles   5.   10.5 mph

6.   4 min   7.   B   8.   A   9.   A